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MONTE CARLO RELIABILITY ANALYSIS(U) NORTHWESTERN UNIV
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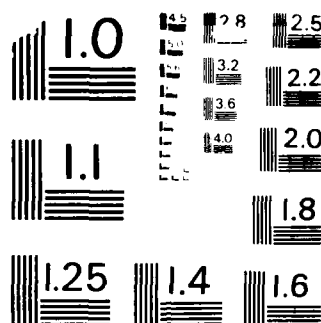
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DOCUMENTATION PAGE

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19. ABSTRACT (Continue on reverse if necessary and identify by block number) <p>The work carried out during the 1986/87 contract year is summarized: The Monte Carlo simulation methods are generalized to treat nonMarkovian systems and applied to problems with time and batch replacement maintenance policies. Trimodular redundant systems with reconfigurable spares are also simulated. A new graphical technique is developed for the presentation of Monte Carlo results. Finally, an investigation of the reliability behavior of brittle solids is initiated.</p>																
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Accomplishments: During the project year ending August 31, 1987 there were three areas in which work was carried out and results obtained. These relate to nonMarkovian extensions of the Markov Monte Carlo simulation methods, graphical presentations of Monte Carlo results, and prediction of time to failure distributions of brittle components.

We have completed the implementation of the generalization of our earlier inhomogeneous Markov Monte Carlo code to treat parts replacement problems for which the Markov property is lost. The resulting method retains the use of the self-transition sampling¹ to model time-dependent failure rates and is fully compatible with our variance reduction techniques.

The effectiveness of the new techniques have been demonstrated by applying them to two classes of problems. In the first, comparisons are made between batch replacement and time replacement policies on redundant configurations of components. This work was recently presented at an international topical meeting on probabilistic risk assessment; a reprint is included as an appendix.² In addition, we have used our code to make reliability simulations of a widely used redundant configuration for flight-critical avionics systems: the trimodular redundant (TMR) system with reconfigurable spares. This work³ will be reported at the Annual Reliability and Maintainability Symposium to be held in January, 1988.

The second area was unanticipated at the time the proposal was written. It is the development of an effective means for the graphical presentation of Monte Carlo results. An argument in favor of analytical or deterministic numerical methods for the analysis of Markov processes has been that from them one obtains results in the form of time-dependent curves, while Monte Carlo Simulation yields only a single result at a specified time. Since a great deal of insight into the nature of the solution is lost, Monte Carlo is often relegated to a method of last resort for otherwise intractable problems. We have circumvented this problem by treating the Monte Carlo simulation as a set of grouped life-test data and employing nonparametric methods to generate curves of reliability or availability vs time. The resulting techniques increase the computation times over those required for a result at a specified time by only a few percent. They were employed to generate the curves shown in the appendix and will be reported in a short paper.⁴

The third area involves the development of methods to generate the time-dependent failure rate curves needed to estimate wear or aging effects in Monte Carlo or deterministic treatments of reliability problems. We are focusing our efforts on the mathematical representation of fatigue failures of brittle mechanical components. We have tentatively constructed a model in which finite element results can be represented as probability density functions of stress which in turn can be incorporated into Monte Carlo reliability simulations.

Personnel: In addition to the principal investigator, the contract continued to support a graduate student, Franz Boehm, who is seeking the PhD in mechanical engineering. In addition, the research was assisted by an MS student, Mr. Uve Hald, who received no support from the AFOSR contract.

Travel: During the year the principal investigator made a one day visit to the Rome Air Development Center to confer with personnel in the Reliability and Maintainability Section, and he attended the Annual Reliability and Maintainability Symposium. These visits were instrumental in bringing about the work on the TMR system described above. In addition, part of the principal investigator's summer appointment at the University of Stuttgart was spent conferring with Dr. Lauf and others in developing the third area listed above.

References

1. E. E. Lewis and Z. Tu, "Monte Carlo Reliability Modeling by Inhomogeneous Markov Processes," Reliability Engr. 16, 277-296 (1986).
2. E. E. Lewis, F. Boehm, U. P. Hald, and Z. Tu, "Generalization of Markov Monte Carlo Reliability Analysis to Include Non Markovian Maintenance Strategies," Trans. Int. Topical Conf. on Probabilistic Safety Assessment and Risk Management, Zurich, Aug. 30-Sept. 4, 1987.
3. Franz Boehm, Uve P. Hald and E. E. Lewis, "Parts Removal in Continuous Time Monte Carlo Reliability Simulation," Ann. Reliab. & Maint. Symp., 1988 (awaiting publication).
4. E. E. Lewis, Z. Tu, U. P. Hald and F. Boehm, "Explicit Time Dependent Analysis in Monte Carlo Reliability Simulation," Trans. Am. Nucl. Soc. (awaiting publication).

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APPENDIX A

From Proc. Probabilistic Safety Assessment and Risk Management, I, TUV, Rheinland 1987.

GENERALIZATION OF MARKOV MONTE CARLO RELIABILITY ANALYSIS TO INCLUDE NONMARKOVIAN MAINTENANCE STRATEGIES

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Northwestern University, Evanston, IL 60201, USA]

ABSTRACT

The Lagrangian approach to Markov Monte Carlo methods for systems reliability analysis is generalized to include nonMarkovian phenomena in which system components are replaced. The method is then employed to analyze the unreliability and unavailability of a number of redundant systems in which maintenance is carried out by batch or time replacement of aging components.

INTRODUCTION

The Lagrangian approach to Markov Monte Carlo methods has been shown to be very effective for estimating reliability and availability of complex systems. The ability to treat general component dependencies in multicomponent systems, coupled with the use of variance reduction techniques to greatly increase sampling efficiency, results in highly efficient algorithms, capable of treating Markov models that would be intractable by deterministic computational methods.^{1,2} More recently, the Monte Carlo formulation has been generalized through a nonanalog sampling technique called self-transitions to treat time-inhomogeneous Markov processes.³ This has allowed the replacement of constant failure rates with more realistic "bathtub" curves thereby permitting the simulation of component wear and periodic preventive maintenance.

In a variety of problems, some critical to reactor safety, departures from Markov models are required. For, if as-good-as-new repair or parts replacement are permitted following revealed failures, the Markov property⁴ is lost. This is illustrated by the failure rate curves in Fig. 1. The solid line (curve c) represents the failure rate (with preventive maintenance) in a time-inhomogeneous Markov calculation. Curve (c) is a reasonable approximation to the as-good-as-old repair (curve b) since the time between failure and repair ($t_r - t_f$) normally is small. However, for as-good-as-new repair (curve a) faithful modeling requires that the failure rate curve be reinitialized at t_r . Moreover, if age (as opposed to batch) replacement policies are to be studied, the times at which preventive replacement is carried out then also depend on the time of the last component failure.

In this paper earlier work in applying Monte Carlo techniques to the evaluation of Markov reliability models¹⁻³ is generalized to systems in which the Markov property must be violated in order to retain the age of each replaceable component in the simulation. For only in this way can classes of reliability problems that combine component wear, preventive maintenance, and parts replacement be treated. Such analysis is required, for example, to

determine the effects of alternative maintenance policies on the reliability and availability of highly redundant nuclear safety systems.

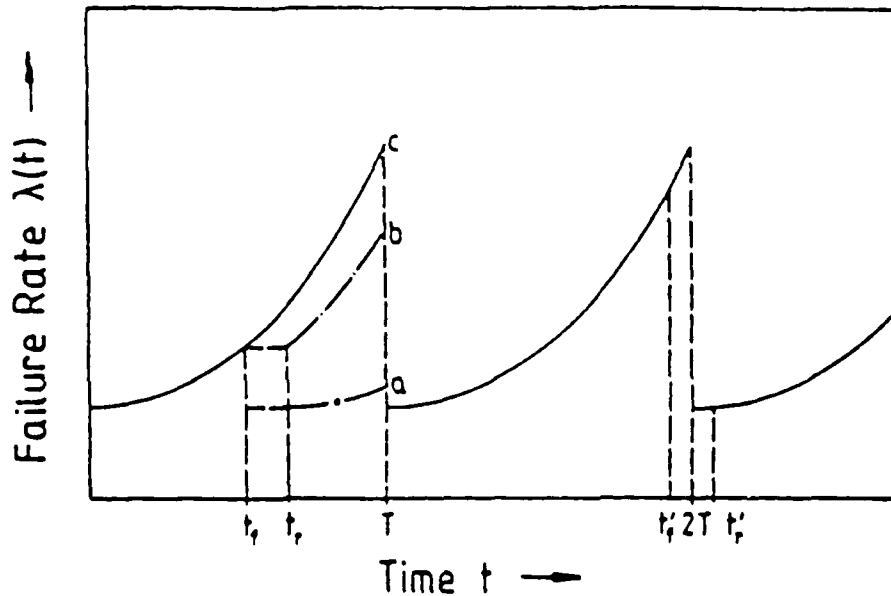


Figure 1: Failure rate curves showing three models for repair of revealed failures: (a) as-good-as new; (b) as-good-as old; (c) continuous wear.

THEORY

The generalization of the Monte Carlo formalism to treat nonMarkovian renewal processes can be most compactly summarized by retaining the framework used in simulating reliability problems represented as continuous-time, inhomogeneous Markov processes. Let $p_k(t)$ be the probability that a system is in state k at time t , where each of the 2^n states for an n -component system constitutes a unique combination of operating and failed components. The equations to be simulated are then

$$\frac{d}{dt} p_k(t) = -\gamma_k(t)p_k(t) + \sum_{k'} q(k, k', t) \gamma_{k'}(t) p_{k'}(t), \quad (1)$$

with initial conditions $p_k(0) = \delta_{k0}$. The transition rate $\gamma_k(t)$ out of state k is given by

$$\gamma_k(t) = \sum_{l \in O_k} \lambda_{lk}(t) + \sum_{l \in F_k} \mu_{lk}(t), \quad (2)$$

where $\lambda_{lk}(t)$ and $\mu_{lk}(t)$ are the failure and repair rates of component l , while in state k and O_k and F_k are the sets of operational and failed components, respectively, in state k . The quantity $q(k, k', t)$ is the conditional probability that given a transition out of state k at time t , the new state will be k' ; it may be written as

$$q(k|k',t) = \gamma_{kk'}(t)/\gamma_{k'}(t), \quad (3)$$

where $\gamma_{kk'}(t)$ is the component failure or repair rate that corresponds to the $k' \rightarrow k$ transition.

In Markov Monte Carlo each of the N trials consists of following the state transitions through some finite period of time, say the design life of the system, T . From time t' and state k' , the time of the next transition is sampled from the cumulative probability distribution

$$F(t|t',k') = 1 - \exp\left\{-\int_{t'}^t \gamma_{k'}(t'')dt''\right\}. \quad (4)$$

In analog Monte Carlo simulation of homogeneous Markov processes, in which the failure rates are constant, the time can be sampled using a uniformly distributed random number in the direct inverse method. The sampling is modified by the use of self transitions if it is necessary to treat the time-dependent failure rates that appear with wear or early failures³. For computational efficiency nonanalog variance reduction normally is employed. In this, the sampling distribution of Eq. (4) is modified to force more transitions, and each trial then carries a weight which is appropriately altered to maintain unbiased estimates of the system unreliability or unavailability. After each transition a second random number is generated to sample Eq. (3) and determine the new system, state k . Once again, nonanalog variance reduction is employed to enhance the number of failures and suppress the number of repairs. The resulting biasing of the state transition matrix is compensated once again by a change in the trial weight to maintain an unbiased reliability or availability estimate¹.

To determine when system failure has resulted from state transition, a fault tree describing the component configuration is evaluated qualitatively either by bottom up evaluation or by using cut sets. The tally for the unreliability is

$$\hat{U}_r = \frac{1}{N} \sum_{t_n < T} w_n \quad (5)$$

where w_n is the weight of the n th trial at the time of system failure, if system failure for that trial occurs at $t_n < T$. The corresponding mission availability estimator is

$$\hat{U}_a = \frac{1}{TN} \sum_{n=1}^N w_n \Delta_n \quad (6)$$

where w_n is the history weight at the time of the first system failure. Since in our algorithms the calculation reverts to analog Monte Carlo after the first system failure in a trial, the weight at the time, t_n , of the first failure is multiplied by Δ_n the total down time for the duration of the trial. For both unreliability and unavailability the same variance is tallied along with Eqs. (5) or (6), and the central limit theorem is used to estimate 68% confidence intervals for the results.

Component replacement of renewal is incorporated into the above formalism by replacing t with a vector $\underline{\tau}$ in the transition rates and probabilities appearing in the Markov equations. The i th component of the vector $\underline{\tau}$ is just the time since the i th component in the system was replaced, or underwent as-good-as-new maintenance. Equation (2) for the k th state transition rate is thus replaced by

$$\gamma_k(\underline{\tau}) = \sum_{\ell < 0_k} \lambda_{\ell k}(\tau_\ell) + \sum_{\ell \in F_k} \nu_{\ell k} \quad (7)$$

where τ_i is the age of the i th component. Likewise, the transition probability is now given by

$$q(k|k', \underline{\tau}) = \gamma_{kk'}(\underline{\tau}) / \gamma_k(\underline{\tau}) .$$

The ability to track Monte Carlo trials in which the age of each component must be incorporated into the transition probabilities has been incorporated into our Monte Carlo simulations. Moreover, this generalization placed no limitations on the use of presently-used importance sampling techniques or on the use of the self-transition technique for treating time dependent failure rates.

RESULTS

To demonstrate the use of the component renewal feature of Monte Carlo reliability analysis simulations have been made for a number of redundant configurations in which component wear is present, and/or in which either time or batch replacement is used as maintenance policies. Recall that in time replacement a component is replaced at failure or after it has been in operation for a predetermined length of time, whichever occurs first; in batch replacement the component is replaced at failure or during predetermined maintenance times that do not depend on how long the component has been in operation.

In Tables I and II the unreliability and unavailability results for four different systems, namely a single component and (1/2), (1/3) and (2/3) active parallel systems. The components are taken to be identical; their failure rates are represented by Weibull distributions, i.e.

$$\lambda(t) = \lambda_0 + \frac{m}{\theta} (t/\theta)^{m-1},$$

with the parameters $\lambda_0 = 0.013/\text{yr}$, $m = 2.5$ and $\theta = 7.5 \text{ yr}$. The repair rate is given as $\nu = 10/\text{yr}$. The design life of the systems is 5 yr. The time of the first maintenance for the single component is $t = 1 \text{ yr}$. For the multicomponent systems, maintenance is performed on a staggered basis, i.e., for the two component system the time of the first maintenance of component 1 is $t = 1 \text{ yr}$ and $t = 2 \text{ yr}$ for component 2. For the three component system the time of the first maintenance is $t = 0.667 \text{ yr}$ for component 1, $t = 1.333 \text{ yr}$ for component 2 and $t = 2 \text{ yr}$ for component 3. The maintenance intervals in case of batch replacement and the age of the component in case of time replacement are taken to be $\Delta t = 2 \text{ yr}$ for all calculations.

TABLE I: Unreliabilities for example problems.

System	Model ¹	Without maintenance	With maintenance
1/1	1	$(0.3428 \pm 0.0027) * 10^0$	$(0.1350 \pm 0.0008) * 10^0$
	2	$(0.3428 \pm 0.0027) * 10^0$	$(0.1350 \pm 0.0008) * 10^0$
	3	$(0.3428 \pm 0.0027) * 10^0$	$(0.1350 \pm 0.0008) * 10^0$
1/2	1	$(0.9259 \pm 0.0175) * 10^{-2}$	$(0.7582 \pm 0.0066) * 10^{-3}$
	2	$(0.6886 \pm 0.0131) * 10^{-2}$	$(0.7457 \pm 0.0066) * 10^{-3}$
	3	$(0.6886 \pm 0.0131) * 10^{-2}$	$(0.7578 \pm 0.0067) * 10^{-3}$
1/3	1	$(0.1816 \pm 0.0086) * 10^{-3}$	$(0.2990 \pm 0.0039) * 10^{-5}$
	2	$(0.1114 \pm 0.0044) * 10^{-3}$	$(0.2899 \pm 0.0039) * 10^{-5}$
	3	$(0.1114 \pm 0.0044) * 10^{-3}$	$(0.3008 \pm 0.0041) * 10^{-5}$
2/3	1	$(0.2747 \pm 0.0085) * 10^{-1}$	$(0.2268 \pm 0.0016) * 10^{-2}$
	2	$(0.2080 \pm 0.0068) * 10^{-1}$	$(0.2227 \pm 0.0016) * 10^{-2}$
	3	$(0.2080 \pm 0.0068) * 10^{-1}$	$(0.2257 \pm 0.0016) * 10^{-2}$

Table II: Interval unavailabilities for example problems.

System	Model ¹	Without Maintenance	With Maintenance
1/1	1	$(0.1200 \pm 0.0008) * 10^{-0}$	$(0.2756 \pm 0.0045) * 10^{-2}$
	2	$(0.7229 \pm 0.0094) * 10^{-2}$	$(0.2714 \pm 0.0038) * 10^{-2}$
	3	$(0.7229 \pm 0.0094) * 10^{-2}$	$(0.2780 \pm 0.0039) * 10^{-2}$
1/2	1	$(0.9397 \pm 0.0174) * 10^{-4}$	$(0.7721 \pm 0.0169) * 10^{-5}$
	2	$(0.7022 \pm 0.0152) * 10^{-4}$	$(0.7520 \pm 0.0166) * 10^{-5}$
	3	$(0.7022 \pm 0.0152) * 10^{-4}$	$(0.7665 \pm 0.0170) * 10^{-5}$
1/3	1	$(0.1255 \pm 0.0041) * 10^{-5}$	$(0.2053 \pm 0.0073) * 10^{-7}$
	2	$(0.7354 \pm 0.0299) * 10^{-6}$	$(0.1981 \pm 0.0072) * 10^{-7}$
	3	$(0.7354 \pm 0.0299) * 10^{-6}$	$(0.2030 \pm 0.0074) * 10^{-7}$
2/3	1	$(0.2728 \pm 0.0054) * 10^{-3}$	$(0.2292 \pm 0.0045) * 10^{-4}$
	2	$(0.2023 \pm 0.0044) * 10^{-3}$	$(0.2241 \pm 0.0045) * 10^{-4}$
	3	$(0.2023 \pm 0.0044) * 10^{-3}$	$(0.2300 \pm 0.0046) * 10^{-4}$

¹ Model 1 ... continuous aging with batch replacement,
 Model 2 ... as good as new repair with batch replacement, with maintenance
 Model 3 ... as good as new repair with time replacement.

The data in Tables I and II are indicative of the increases in reliability and availability through component maintenance and replacement. Equally valuable is the ability to examine the time dependence of the reliability and/or availability. To this end, algorithms have been developed which will allow the generation of the time dependent quantities along with the corresponding confidence interval. This is done while adding less than 10% to the computing time of a Monte Carlo Simulation⁶.

In Figures 2 and 3 are shown the unreliability and interval unavailability vs time for the 1/2 active parallel systems. Each run is for 1000 trials. Three sets of curves are shown, each with three lines corresponding to the estimator and the 68% confidence interval.

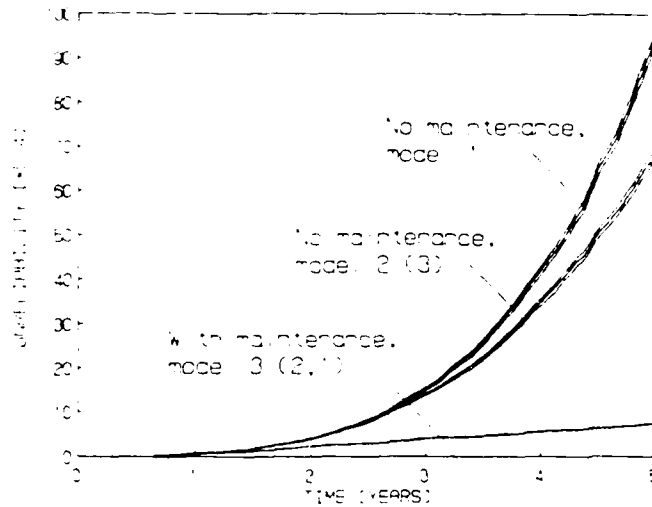


Figure 2: Unreliability versus time.

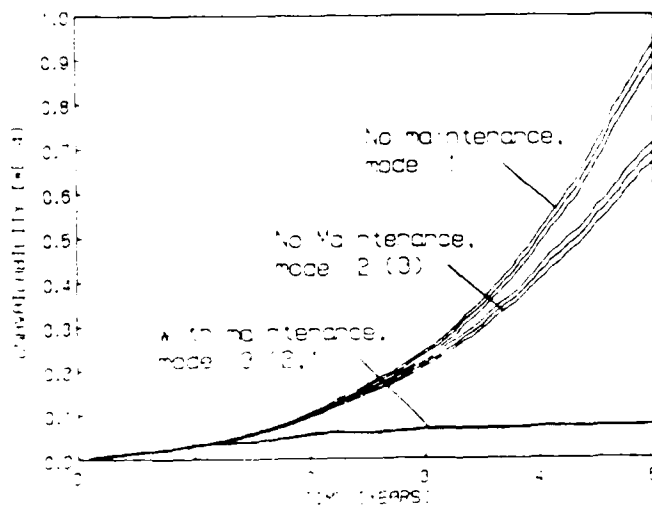


Figure 3: Interval unavailability versus time.

The highest unreliability and unavailability are for model (1) with no maintenance. The results for model 2 and model 3 with no maintenance are nearly indistinguishable and are hence shown as one set of curves. Finally the smallest unreliabilities and maintainabilities shown in Figs. 2 and 3 are for the maintained systems. Here the differences between the three models are nearly indistinguishable. As could be expected, the curves for the unmaintained systems are concave up and demonstrating the marked effects of component aging. Where maintenance is present the unreliability uses in a more linear manner while the unavailability levels off toward an asymptotic value.

Similar results are obtained for standby configurations and for multi-component systems with more complex redundant configurations. Brevity requires that they not be presented here.

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